

# On the pulsation mode identification of short-period Galactic Cepheids

G. Bono<sup>1</sup>, W.P. Gieren<sup>2</sup>, M. Marconi<sup>3</sup>, and P. Fouqué<sup>4</sup>

1. Osservatorio Astronomico di Roma, Via Frascati 33, 00040 Monte Porzio Catone, Italy;

Visiting Astronomer, ESO/Santiago, Chile; [bono@coma.mporzio.astro.it](mailto:bono@coma.mporzio.astro.it)

2. Dept. de Física, Grupo de Astronomia, Univ. de Concepcion, Casilla 160-C,

Concepcion, Chile; [wgieren@coma.cfm.udec.cl](mailto:wgieren@coma.cfm.udec.cl)

3. Osservatorio Astronomico di Capodimonte, Via Moiariello 16, 80131 Napoli, Italy;

[marcella@na.astro.it](mailto:marcella@na.astro.it)

4. Observatoire de Paris-Meudon, DESPA F-92195 Meudon Cedex, France; and ESO,

Casilla 19001, Santiago 19, Chile; [pfouque@eso.org](mailto:pfouque@eso.org)

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## ABSTRACT

We present new theoretical Period-Radius (PR) relations for first overtone Galactic Cepheids. Current predictions are based on several sequences of nonlinear, convective pulsation models at solar chemical composition ( $Y=0.28$ ,  $Z=0.02$ ) and stellar masses ranging from 3.0 to 5.5  $M_{\odot}$ . The comparison between predicted and empirical radii of four short-period Galactic Cepheids suggests that QZ Nor and EV Sct are pulsating in the fundamental mode, whereas Polaris and SZ Tau pulsate in the first overtone. This finding supports the mode identifications that rely on the comparison between direct and Period-Luminosity (PL) based distance determinations but it is somewhat at variance with the mode identification based on Fourier parameters. In fact, we find from our models that fundamental and first overtone pulsators attain, for periods ranging from 2.7 to 4 d, quite similar  $\phi_{21}$  values, making mode discrimination from this parameter difficult. The present mode identifications for our sample of Cepheids are strengthened by the accuracy of their empirical radius estimates, as well as by the evidence that predicted fundamental and first overtone radii do not show, within the current uncertainty on the Mass-Luminosity (ML) relation, any degeneracy in the same period range. Accurate radius determinations are therefore an excellent tool to unambiguously determine the pulsation modes of short-period Cepheids.

*Subject headings:* stars: Cepheids – stars: evolution – stars: fundamental parameters – stars: oscillations

## 1. Introduction

Classical Cepheids are an important link between stellar and extragalactic research, and they are widely adopted not only to estimate cosmic distances (Feast 1999; Gieren, Fouqué, & Storm 2000) but also to investigate young stellar populations in external galaxies (Magnier et al. 1997; Macri et al. 2001). Given the extreme usefulness of Cepheids for various astrophysical fields, it is crucial to establish their physical properties with high accuracy. Even though both evolutionary and pulsational properties of Cepheids are based on robust theoretical predictions we are still dealing with several long-standing unsolved problems, such as the universality of both PL and Period-Luminosity-Color (PLC) relations (Bono et al. 1999a; Gieren et al. 1999; Caputo et al. 2000; Groenewegen 2000), and the explanation of the Fourier parameters of the Bump Cepheids (Feuchtinger, Buchler, & Kollath 2000). Moreover, the large photometric databases collected by the microlensing experiments (EROS, MACHO, OGLE) have provided the unique opportunity to investigate the occurrence of exotic objects in the Magellanic Clouds, such as mixed-mode Cepheids. At the same time, the excellent light curve phase coverage and the large sample of Cepheids measured by these projects gave also the opportunity to increase, by more than one order of magnitude, the number of detected first overtone (FO) Cepheids, and to settle (Beaulieu et al. 1995; Welch et al. 1995) the long-standing problem (Pel & Lub 1978; Gieren 1982; Böhm-Vitense 1988) concerning the occurrence of this mode among classical Cepheids.

One of the main results of these investigations was that fundamental (F) and FO Cepheids are distributed, as expected, along two distinct sequences in the magnitude-period plane and present quite different Fourier parameters in certain period ranges. The latter is a crucial finding for two different reasons: i) classical Cepheids, at variance with RR Lyrae stars, cannot be easily split into F and FO variables according to the shape of their light curves. ii) the result strongly supports the evidence originally brought forward by

Antonello, Poretti & Reduzzi (1990), based on the light curve Fourier parameters, that the so-called s-Cepheids pulsate in the first overtone. This indication was further strengthened by the evidence that the Fourier parameters of Cepheid radial velocity curves do show a very similar behavior (Kienzle et al. 1999; Moskalik et al. 2001) as their light curve counterparts. However, even though the results disclosed by the massive microlensing data can hardly be questioned, it is worth mentioning that by comparing directly measured distances based on the infrared surface brightness method and distance estimates based on the PL relation, Gieren, Fouqué, & Gomez (1997, 1998; hereinafter GFG97, GFG98) found evidence that at least some short-period, s-Cepheids could rather be F than FO pulsators. The overall scenario is jazzed up by the fact that current Cepheid models do not account for the Fourier parameters of observed light and radial velocity curves (Antonello & Aikawa 1995; Feuchtinger et al. 2000). In view of these facts, it is desirable to have additional quantitative criteria which allow us a firm identification of Cepheid pulsation modes.

Parallel to the mentioned studies, several investigations in the recent literature have been devoted to the comparison between theoretically predicted, and empirical measurements of Galactic Cepheid radii (Laney & Stobie 1995; Bono, Caputo, & Marconi 1998, hereinafter BCM98; Riipei et al. 1998; Gieren et al. 1999; Nordgren et al. 2000). Most of this work has focused on Cepheids pulsating in the fundamental mode, since these variables are characterized by larger luminosity and radial velocity amplitudes when compared to first overtones which makes it easier to obtain accurate radii for them, particularly when Baade-Wesselink type techniques of radius determination are employed. However, thanks to optical interferometric measurements Nordgren et al. (2000) recently succeeded in determining the mean angular diameter of the nearest Cepheid,  $\alpha$  UMi (Polaris), from which they were able to estimate its radius. The pulsation behavior of this Cepheid is somewhat peculiar (Kamper & Fernie 1998) but it has been recently classified as a first overtone (Feast & Catchpole 1997). In order to compare the new radius

evaluation with both empirical and theoretical PR relations, Nordgren et al. were forced to fundamentalise the period of Polaris, i.e. they added 0.148 to its logarithmic period, since at present we still lack both empirical and theoretical PR relations for first overtones.

The main aim of this Letter is to present a new theoretical PR relation for FO Galactic Cepheids constructed by adopting a fixed chemical composition ( $Y=0.28$ ,  $Z=0.02$ ) and several stellar masses ranging from 3.0 to 5.5  $M_{\odot}$ , and exploit this relation as a new tool for discriminating F and FOs among classical Cepheids. To account for current uncertainties on the predicted luminosity of intermediate-mass Cepheids, the models were constructed by adopting two different ML relations. In §2 we discuss current empirical evidence concerning the mode identification of short-period Cepheids. In §3 we compare the predicted PR relations with observed Cepheid radii. Conclusions about the mode identification from measured radii, and a few comments on the developments of this project close the paper.

## 2. Empirical facts and theoretical predictions

To assess the reliability of the mode identification for classical Cepheids based on their Fourier parameters, we selected four short-period Galactic Cepheids, namely EV Sct, SZ Tau, QZ Nor, and Polaris as test objects. These variables were selected because they have accurate, period-independent individual distance and radius estimates from the infrared surface brightness technique (GFG97,98) and trigonometric parallaxes (Nordgren et al. 2000), respectively, and all of them are generally considered to be FOs based on their short-periods and low-amplitude, nearly sinusoidal light curves. Also, they have accurate spectroscopic and photometric data and Table 1 summarizes their key empirical observables.

We already mentioned that Fourier parameters are widely adopted in the literature to identify the pulsation mode, and therefore we evaluated the Fourier parameters  $R_{21}$  and

$\phi_{21}$  for the selected Cepheids. We estimated these parameters from the observed radial velocity curves, to avoid possible subtle errors affecting the transformation of the theoretical light curves onto the observational plane. Fig. 1 shows from top to bottom the Fourier parameters  $R_{21}$ ,  $\phi_{21}$ , and  $A_1$  of the four Cepheids together with the Fourier parameters for FO and F Galactic Cepheids by Kienzle et al. (1999) and Moskalik et al. (2001). A glance at the data plotted in this figure shows quite clearly that the Fourier parameters of our sample are indeed typical for Cepheids classified as first overtones.

An independent observable we can adopt to assess whether a star is pulsating in the F or in the FO mode is the stellar radius. As a matter of fact, a star pulsating in the FO is brighter and larger than a star pulsating with the same period in the fundamental mode. Owing to the lack of empirical PR relations for FO Cepheids, we decided to use theoretical radii predicted by nonlinear models to perform this test for our four stars. New PR relations for fundamental mode Cepheids based on nonlinear, convective models were recently provided by BCM98. Even though the idea to compare predicted and empirical radii by artificially decreasing the fundamental period of the current theoretical PR relation is viable, it is quite risky. The reason is twofold: i) the width in temperature of the instability region where the F mode is unstable is on average larger than for the FO. As a consequence, the comparison between fundamentalised variables and truly fundamental PR relations can hardly be adopted to constrain the pulsation mode; ii) recent observational data (EROS, OGLE) on Magellanic Cepheids seem to support the evidence that the slope of the fundamental PL relation changes at very short periods (Bauer et al. 1999; Groenewegen 2000). Therefore the PR relations for F and FO Cepheids could have different slopes.

To avoid these complications we have constructed several new sequences of nonlinear, convective models by adopting a chemical composition typical of solar neighborhood Cepheids, i.e.  $Y=0.28$ ,  $Z=0.02$ . In order to assess on a quantitative basis the difference, if

any, between F and FO Cepheid PR relations we adopted the same theoretical framework as adopted by BCM98 and by Bono, Marconi, & Stellingwerf (1999b). The reader interested in the details of the input physics is referred to these papers. Moreover, to account for the difference in the luminosity of intermediate-mass Cepheids predicted by evolutionary models that include (noncanonical) or neglect (canonical) convective core-overshooting during hydrogen burning phases, we adopted, according to BCM98, two different ML relations. The mass/luminosity values adopted for canonical models are:  $M/M_{\odot}$ - $\log L/L_{\odot}$ =3.5-2.51, 4.0-2.72, 4.5-2.90, 5.0-3.97, and 5.5-3.07; while for the noncanonical models are: 3.0-2.52, 4.0-2.97, 4.6-3.19, and 4.75-3.24. In the former set the effective temperature of stable FO models ranges from 5800 to 6500 K, and from 5650 to 6200 K in the latter one. On the basis of these calculations we derived the following canonical and noncanonical PR relations:

$$\log R = 1.250(\pm 0.005) + 0.755(\pm 0.007) \log P \quad \sigma = 0.005$$

$$\log R = 1.219(\pm 0.004) + 0.737(\pm 0.005) \log P \quad \sigma = 0.004$$

where  $R$  is the radius (solar units),  $P$  the period (d), and  $\sigma$  is the standard deviation. The pulsation properties of these models will be discussed in a forthcoming paper. The previous linear regressions bring out that the intrinsic dispersion of the FO PR relations is a factor of four smaller than for the F PR relations (0.005 against 0.02) derived by BCM98. Note also that the slopes of FO relations are significantly steeper than the one for F pulsators ( $\approx 0.75$  against  $\approx 0.65$ ). This difference is caused by two different effects: the FO instability region is both systematically narrower, and bluer than the F one. This means that FOs are, at fixed period, systematically brighter than F pulsators. These findings provide a clear argument that one should use *pure* F and FO PR relations to constrain the pulsation mode.

Fig. 2 shows the comparison of predicted Fourier parameters for both F (filled triangles) and FO pulsators (open circles) with those of our selected short-period Cepheids. The outcome of the comparison between theory and observations is not very comfortable,

and indeed current models seem to suggest that for periods ranging from 2.7 to 4 d both F and FO Cepheids attain similar  $\phi_{21}$  values. The velocity amplitude  $A_1$  and  $R_{21}$  seem to be less affected by this degeneracy problem, since F pulsators attain values that are, at fixed period, larger than FO ones. The sequences of F and FO pulsators we constructed are too coarse to quantitatively assess the nature of the  $\phi_{21}$  jump located at  $P \approx 4.2$  d (see Fig. 1). As a consequence, this finding is only a preliminary hint and more models are necessary to establish the extent of this degeneracy. Nevertheless it seems to be clear that, in the 3-4 d period range, the mode identification based on  $\phi_{21}$  values might be quite uncertain.

To shed new light on this problem we decided to investigate whether empirical radius measurements can be adopted to disentangle the mode identification problem. Fig. 3 shows the comparison of theoretical PR relations for both FO (solid and dotted lines) and F (BCM98, dashed and dashed-dotted lines) pulsators with empirical data for the selected objects. The data plotted in this figure disclose several interesting results. At odds with the mode identification based on the Fourier parameters, the comparison between predicted and observed radii yield strong evidence that both EV Sct and QZ Nor are fundamental pulsators. This result seems quite robust for three reasons: i) predicted F and FO radii are, in this period range, substantially different; ii) the empirical radii have quite small uncertainties (both random and systematic). iii) the predicted dependence on metallicity is marginal in this period range. In fact, a decrease in the metal content from  $Z=0.02$  to  $Z=0.008$  causes, according to BCM98, a decrease in  $\log R$  of approximately 0.02 dex. Moreover, FO radii predicted by Bono, Castellani, & Marconi (2000) for  $Z=0.008$  attain, at fixed period, values that are in very good agreement with FO radii at solar metallicity. Thus suggesting that FO radii for  $0.008 \leq Z \leq 0.02$  marginally depend on metallicity.

As far as Polaris is concerned, its radius provides evidence that this object is indeed pulsating in the first overtone, as suggested by Feast & Catchpole (1997) on the grounds



of the Hipparcos distance estimate, although this conclusion is somewhat weakened by the current observational uncertainties. A detailed analysis of both light and radial velocity curves (Kamper & Fernie 1998) suggest that Polaris is currently undergoing a transient phase during which both pulsation amplitudes and period are rapidly changing. Therefore, further accurate parallax measurements are needed to improve the accuracy of its radius determination. The empirical radius determination for SZ Tau seems to suggest that this object is also pulsating in the first overtone mode. We note here that to avoid any systematic uncertainty in the radius determination of this star due to the known variation of its pulsation period (Szabados 1977), we adopted the radius value based on the J-K solution in GFG97, since J and K data were collected simultaneously and do not present a potential phase misalignment problem as do the noncontemporaneous V and K data.

Summarizing, the comparison between theoretically predicted and measured radii is unveiling that among the selected short-period Cepheids two objects are almost certainly fundamental mode pulsators, in contrast to the mode identification based on their Fourier parameters. The data plotted in Fig. 3 show two further interesting results. In contrast with BCM we find that even at short-periods empirical radii are in very good agreement with theoretical predictions. The fundamental mode PR relations present an intrinsic dispersion of the order of 0.02 dex. Therefore empirical radius measurements of F pulsators can be hardly adopted to assess the accuracy of current ML relations. On the other hand, the intrinsic dispersion of FO PR relations is of the order of only 0.005 dex. This suggests that accurate FO radius determinations can supply useful constraints on both evolutionary and pulsational models.

### 3. Conclusions and final remarks

From the current work we find evidence that a comparison of accurately measured empirical Cepheid radii to theoretical radii for F and FO pulsators is a very useful tool to identify the pulsation modes of classical Cepheids. In fact, theoretical predictions based on nonlinear, convective models seem to suggest that F and FO radii are not affected by any degeneracy for periods ranging from 2.7 to 4 days. This is not true for the Fourier parameter  $\phi_{21}$  of radial velocity curves. In fact, two short-period Galactic Cepheids, namely EV Sct, and QZ Nor which on the basis of their Fourier parameters should be classified as FO pulsators are F mode pulsators, as revealed by their radii. The degeneracy between F and FO Fourier parameters is supported by current theoretical predictions. Even though the sequences of models we constructed are relatively coarse in the mass step, the Fourier parameters for fundamental pulsators ( $M = 4.5 \div 5.0 M_{\odot}$ ) based on theoretical radial velocity curves attain values quite similar to FOs with stellar masses ranging from 4.6 to  $5.5 M_{\odot}$ . The radial velocity amplitude and  $R_{21}$  seem to be less affected by this degeneracy.

On the other hand, the comparison between predicted and observed radii strongly suggests that both Polaris and SZ Tau are pulsating in the first overtone, supporting the mode identification suggested by Feast & Catchpole (1997) and by Laney (1997), respectively. In this context it is worth mentioning that our new PR relations for FO Cepheids present an intrinsic dispersion that is approximately a factor of four smaller than that for F PR relations. Moreover and even more importantly, we find that the slope of the former ones is steeper than that for the latter ones, suggesting that the mode identification is more robust if based on *pure* F and FO PR relations. Unfortunately, the number of short-period Cepheids, particularly s-Cepheids, for which accurate radius measurements are currently available is limited to a handful of objects. It goes without saying that new and accurate multiband, near-infrared photometric data and radial velocity measurements

for such objects would be crucial to quantitatively assess the accuracy of the mode identification based on the PR relations. Finally, we remark that FO PR relations are much more sensitive to the ML relation and marginally dependent on the metallicity adopted to construct the pulsation models. Accurate radius measurements for FO Cepheids could therefore supply useful hints on this key relationship predicted by evolutionary models.

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## REFERENCES

- Antonello, E., & Aikawa, T. A&A, 337, 145
- Antonello, E., Poretti, E., & Reduzzi, L. 1990, A&A, 236, 138
- Bauer F., et al. 1999, A&A, 348, 175
- Beaulieu et al. 1995, A&A, 303, 137
- Bersier, D., Burki, G., Mayor, M., & Duquennoy, A. 1994, A&AS, 108, 25
- Böhm-Vitense, E. 1988, ApJ, 324, L27
- Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1999a, ApJ, 512, 711
- Bono, G., Caputo, F., & Marconi, M. 1998, ApJ, 497, L43 (BCM98)
- Bono, G., Castellani, V., & Marconi, M. 2000, ApJ, 529, 293
- Bono, G., Marconi, M., & Stellingwerf, R. F. 1999b, ApJS, 22, 167
- Caputo, F., Marconi, M., Musella, I., & Santolamazza, P. 2000, A&A, 359, 1059
- Feast, M. 1999, PASP, 111, 775
- Feast, M. W., & Catchpole, R. M. 1997, MNRAS, 286, P1
- Feuchtinger, M., Buchler, J. R., & Kollath, Z. 2000, ApJ, 544, 1056
- Gieren, W. P. 1982, PASP, 94, 960
- Gieren, W. P., Fouqué, P., Gomez, M. 1997, ApJ, 488, 74 (GFG97)
- Gieren, W. P., Fouqué, P., Gomez, M. 1998, ApJ, 496, 17 (GFG98)
- Gieren, W. P., Fouqué, P., & Storm, J. 2000, in The VLT Opening Symposium, ESO  
Astrophysics Symposia, eds. J. Bergeron, A. Renzini, (Springer-Verlag: Berlin), 217
- Gieren, W. P., Moffett, T. J., & Barnes, T. G. III, 1999, ApJ, 512, 553
- Groenewegen, M. A. T. 2000, A&A, 363, 901

- Hatzes, A. P., & Cochran, W. D. 2000, *AJ*, 120, 979
- Kamper, K. W., & Fernie, J. D. 1998, *AJ*, 116, 936
- Kienzle, F., Moskalik, P., Bersier, D., & Pont, F. 1999, *A&A*, 341, 818
- Laney, C. D. 1997, in *Half Century of Stellar Pulsation Interpretation: A Tribute to A.N. Cox*, ed. P.A. Bradley & J.A. Guzik, (San Francisco: ASP), 180
- Laney, C. D., & Stobie, R. S. 1995, *MNRAS*, 274, 337
- Macri, L. M., Stanek, K. Z., Sasselov, D. D., & Krockenberger, M. 2001, *AJ*, 121, 870
- Magnier, E. A., Prins, S., Augusteijn, T., van Paradijs, J., & Lewin, W. H. G. 1997, *A&A*, 326, 442
- Metzger, M. R., Caldwell, J. A. R., McCarthy, J. K., Schechter, P. J. 1991, *ApJS*, 76, 803
- Moskalik, P., Krzyt, T., Gorynya, N. A., & Samus, N. N. 2001, in preparation
- Moskalik, P., & Ogloza, W. 2000, in *IAU Colloq. 176, The Impact of Large-Scale Surveys on Pulsating Star Research*, eds. L. Szabados & D. Kurtz (San Francisco: ASP), 237
- Nordgren, T. E., Armstrong, J. T., Germain, M. E., Hindsley, R. B., Hajian, A. R., Sudol, J. J., & Hummel, C. A. 2000, *ApJ*, 543, 972
- Pel, J. W., & Lub, J. 1978, in *IAU Symp. 80, The HR diagram - The 100th anniversary of H.N. Russell*, (Dordrecht: Kluwer), 229
- Ripepi, V., Barone, F., Milano, L., Russo, G. 1997, *A&A*, 318, 797
- Szabados, L. 1997, *CoKon*, 70, 1
- Welch et al. 1995, in *IAU Colloq. 155, Astrophysical Applications of Stellar Pulsation*, ed. R.S. Stobie & Whitelock, P. (San Francisco: ASP), 232

TABLE 1. empirical data<sup>a</sup>

	EV Sct	SZ Tau	QZ Nor	$\alpha$ UMi
Period (d)	3.09097	3.148727	3.78673	3.97267
Radius ( $R/R_{\odot}$ )	$32.5 \pm 0.5$	$45.6 \pm 4.0$	$38.5 \pm 0.5$	$46 \pm 3$
Distance (pc)	$1635 \pm 25$	$692 \pm 61$	$1656 \pm 8$	$132^{+9}_{-8}$
$R_{21}$ (RV)	$0.19 \pm 0.05$	$0.25 \pm 0.03$	$0.16 \pm 0.02$	$0.03 \pm 0.02$
$\phi_{21}$ (RV)	$3.16 \pm 0.37$	$3.48 \pm 0.13$	$3.70 \pm 0.15$	$3.90 \pm 0.38$
$A_1$ (RV)	$7.74 \pm 0.27$	$9.72 \pm 0.17$	$7.38 \pm 0.15$	$0.80 \pm 0.08$

<sup>a</sup> Fourier parameters for EV Sct and SZ Tau were estimated according to data collected by Metzger et al. (1991) and Bersier et al. (1994), while for QZ Nor by Kienzie et al. (1999). Data for  $\alpha$  UMi come from Hatzes & Cochran (2000, period), Kamper & Fernie (1998,  $A_1$ ), Moskalik & Oglozova (2000,  $A_2$ ,  $\phi_{21}$ ), and Nordgren et al. (2000, radius and distance).

Fig. 1.— Top panel: amplitude ratio  $R_{21} \equiv A_2/A_1$ , i.e. the ratio between second and first harmonic amplitude, as a function of period for the sample of fundamental (triangles) and first overtone (open circles) Galactic Cepheids collected by Moskalik et al. (2001), and Kienzie et al. (1999). Data refer to the Fourier fit to the radial velocity curves. The four selected Cepheids were plotted by adopting different symbols. Middle panel: same as the top panel, but for the phase difference  $\phi_{21} \equiv \phi_2 - 2\phi_1$ , i.e. the phase difference between second and first harmonic. Bottom panel: same as the top panel, but for the first harmonic amplitude  $A_1$ . The error bars refer to internal uncertainty in the Fourier fit.

Fig. 2.— Same as Fig. 1, but symbols refer to Fourier parameters of F and FO predicted radial velocity curves. The error bars show the internal uncertainty in the Fourier fit.

Fig. 3.— Comparison between empirical radius determinations and theoretically predicted PR relations at solar chemical composition ( $Y=0.28$ ,  $Z=0.02$ ). Canonical and noncanonical PR relations for first overtone and fundamental pulsators are plotted using different line styles. Symbols are the same as in Fig. 1.